

Astronomers use a technique called *stellar stereography* to determine the density of stars in a star cluster from the observed (two-dimensional) density that can be analyzed from a photograph. Suppose that in a spherical cluster of radius  $R$  the density of stars depends only on the distance  $r$  from the center of the cluster. If the perceived star density is given by  $y(s)$ , where  $s$  is the observed planar distance from the center of the cluster, and  $x(r)$  is the actual density, it can be shown that

$$y(s) = \int_s^R \frac{2r}{\sqrt{r^2 - s^2}} x(r) dr.$$

If the actual density of stars in a cluster is  $x(r) = \frac{1}{2}(R - r)^2$ , find the perceived density  $y(s)$ .

$y(s) =$   